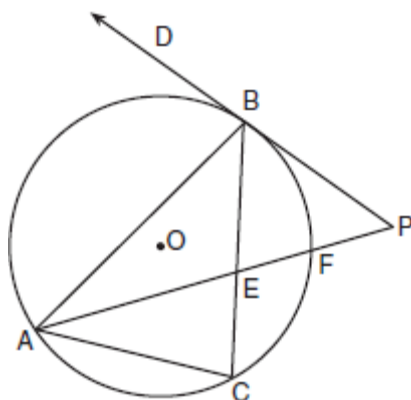Name: _____

## G.G.51: Arcs Determined by Angles: Investigate theorems about the arcs determined by angles intersecting a circle when the vertex is on the circle

1   In the accompanying diagram, $\triangle ABC$ is inscribed in circle O, $\overline{AP}$ bisects $\angle BAC$, $\overrightarrow{PBD}$ is tangent to circle O at B, and $m\angle ACB:m\angle CAB:m\angle ABC = 4:3:2$

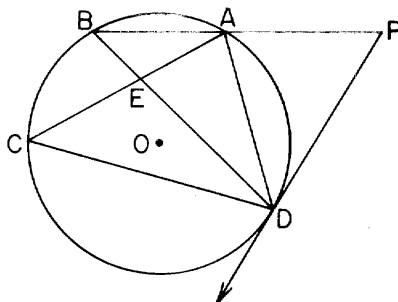Find: $m\angle ABC$, $m\overset{\frown}{BF}$, $m\angle BEP$, $m\angle P$, $m\angle PBC$

2   In the accompanying diagram, $\overline{AB}$ is a diameter of circle O, $\overline{FECA}$ and $\overline{FBG}$ are secants, $m\overset{\frown}{AD}:m\overset{\frown}{DE}:m\overset{\frown}{EB} = 1:3:2$.
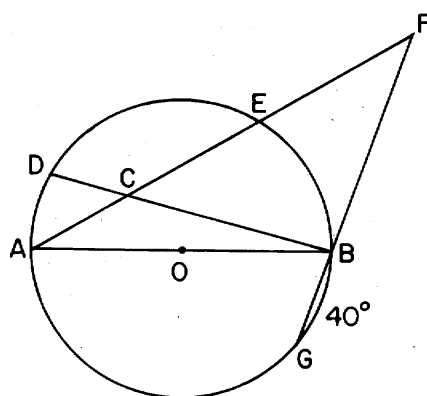
Find $m\overset{\frown}{DE}$, $m\angle ECB$, $m\angle AFG$, $m\angle DBF$, and $m\angle EAB$
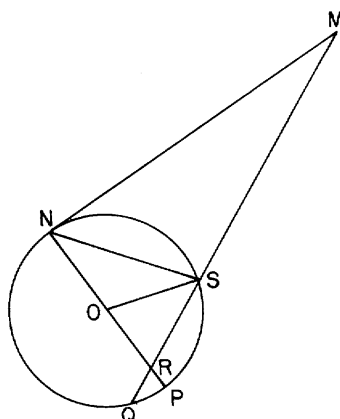
Name: _____

3   In the accompanying diagram, *B* is the midpoint of $\overset{\frown}{AC}$, triangle ADC is inscribed in circle *O*, chords $\overline{AC}$ and $\overline{BD}$ intersect at *E*, $\overrightarrow{PR}$ is a tangent to circle *O* at *D*, $\overline{PAB}$ is a secant, and $m\overset{\frown}{BA}:m\overset{\frown}{AD}:m\overset{\frown}{DC}=2:3:5$.
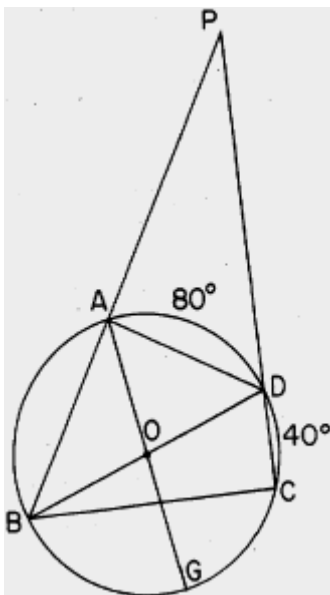

Find: $m\overset{\frown}{BC}$, m∠ADC, m∠AEB, m∠ADP, m∠P

4   In circle *O*, $\overline{MN}$ is a tangent, $\overline{NP}$ is a diameter, $\overline{MQ}$ is a secant, $\overline{OS}$ is a radius, $m\overset{\frown}{QN}=160$, and m∠PNS = 40.

Find $m\overset{\frown}{QP}$, $m\overset{\frown}{PS}$, m∠QRP, m∠NOS, and m∠M
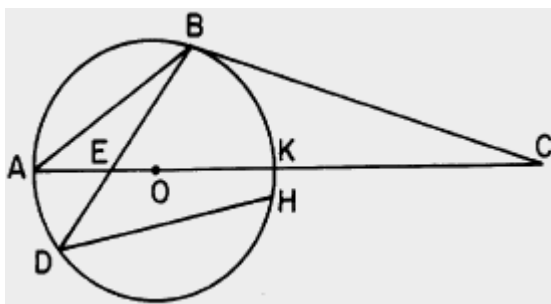
Name: _____

5 Quadrilateral *ABCD* is inscribed in circle *O*, $\overline{BD}$ and $\overline{AG}$ are diameters, $\overline{PAB}$ and $\overline{PDC}$ are secants, $m\widehat{AD} = 80$, and $m\widehat{DC} = 40$.

Find $m\widehat{AB}$, $m\angle BCD$, $m\angle BOG$, $m\angle P$, and $m\angle BAG$
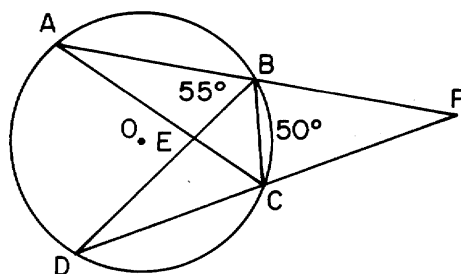
6 Given: circle *O* with $m\widehat{AD}:m\widehat{AB}:m\widehat{BK} = 1:3:2$, diameter $\overline{AK}$ is extended to *C*, $\overline{BC}$ is tangent to circle *O* at *B*, and $\widehat{HK} = 12°$.

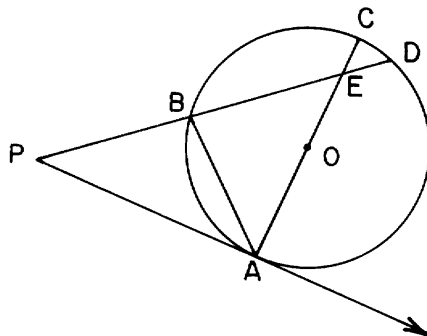Find: $m\widehat{AD}$, $m\angle BCK$, $m\angle BDH$, $m\angle AEB$, $m\angle DBC$

Name: _____

7   In the accompanying diagram of circle O, $\overline{PBA}$ and $\overline{PCD}$ are secants, chords $\overline{AC}$ and $\overline{BD}$ intersect at E, $\overline{BA} \cong \overline{CD}$, chord $\overline{BC}$ is drawn, m$\angle ABD = 55$, and m$\overset{\frown}{BC} = 50$.

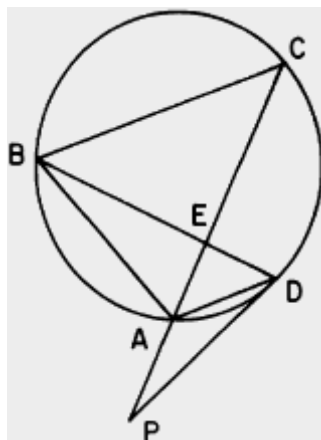Find: m$\angle ACD$, m$\angle P$, m$\angle DBC$, m$\angle AED$, m$\angle PCB$.

8   In the accompanying diagram, $\overrightarrow{PA}$ is a tangent to circle O at point A, secant $\overline{PBD}$ intersects diameter $\overline{AC}$ at point E, m$\angle P = 40$, and m$\overset{\frown}{CD}$:m$\overset{\frown}{DA} = 1:8$.

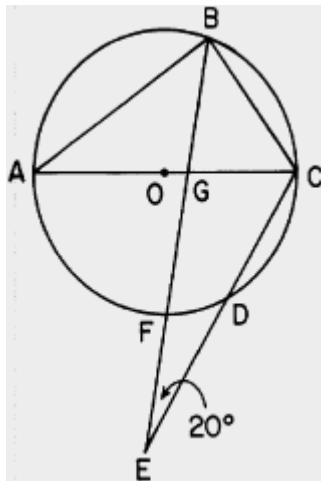Find m$\overset{\frown}{AD}$, m$\overset{\frown}{AB}$, m$\angle BEA$, m$\angle BAC$, and m$\angle PBA$

Name: _____

9   In the accompanying diagram, $\triangle ABC$ is isosceles with $\overline{CB} \cong \overline{CA}$, $m\angle DAC = 45$, $m\overset{\frown}{BC} = 135$, $\overline{PD}$ is tangent to circle O at D, $\overline{PAC}$ is a secant, and chords $\overline{BD}$ and $\overline{AC}$ intersect at E.

Find: $m\overset{\frown}{AD}$, $m\overset{\frown}{AB}$, $m\angle P$, $m\angle ADP$, $m\angle BEC$
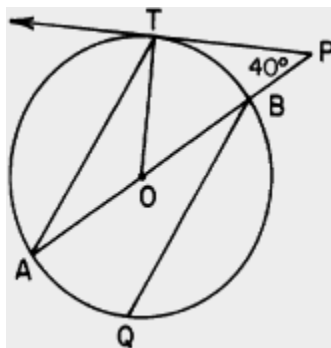
10   In the accompanying diagram, $\triangle ABC$ is inscribed in circle O. Secant $\overline{EFB}$ bisects $\angle ABC$ and intersects diameter $\overline{AOC}$ at G, $\overline{EDC}$ is a secant, $m\angle E = 20$, and $m\overset{\frown}{AB} : m\overset{\frown}{BC} = 3:2$.

Find: $m\overset{\frown}{BC}$, $m\overset{\frown}{FD}$, $m\angle ABE$, $m\angle FGC$, $m\angle ACD$
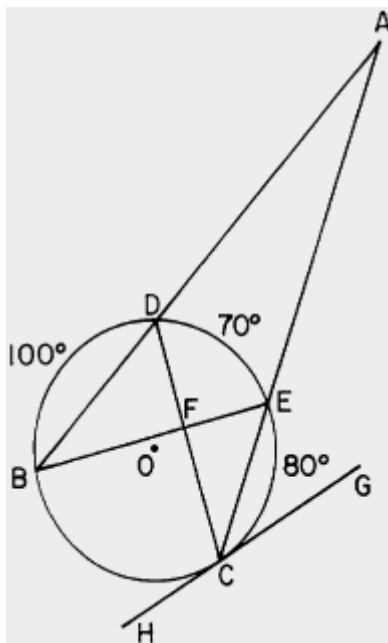
                                                    Name: _____

11 In the accompanying diagram of circle O, $\overline{PBOA}$ is a secant, $\overrightarrow{PT}$ is tangent to circle O at T,
    $m\angle P = 40$, and $\overline{QB} \| \overline{AT}$.

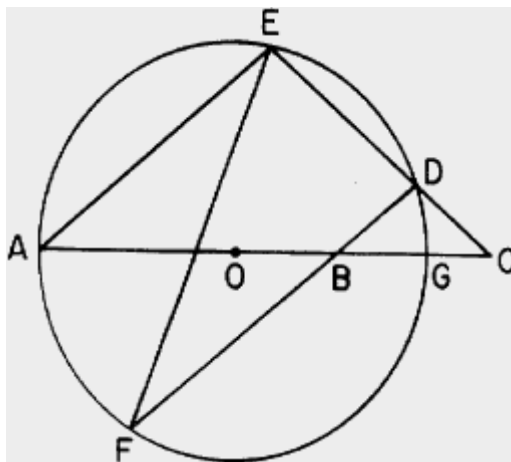   Find: $m\angle BOT$, $m\angle A$, $m\widehat{AT}$, $m\angle ATO$, $m\angle PBQ$

12 In the accompanying diagram of circle O, $\overline{ADB}$ and $\overline{AEC}$ are secants, chords $\overline{BE}$ and $\overline{CD}$
    intersect at F, tangent $\overleftrightarrow{GH}$ intersects circle O at C, $m\widehat{BD} = 100$, $m\widehat{DE} = 70$, and $m\widehat{EC} = 80$.

    Find: $m\angle BAC$, $m\angle BDC$, $m\angle CFE$, $m\angle GCE$, $m\angle AEB$
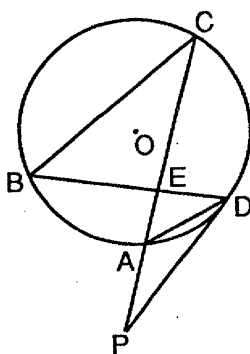
Name: _____

13 In the accompanying diagram of circle O, $\overline{AE}$ and $\overline{FD}$ are chords, $\overline{AOBG}$ is a diameter and is extended to C, $\overline{CDE}$ is a secant, $\overline{AE} \parallel \overline{FD}$, and $m\overset{\frown}{AE}:m\overset{\frown}{ED}:m\overset{\frown}{DG}= 5:3:1$.

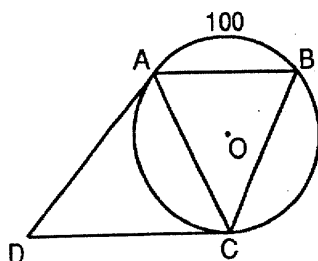Find $m\overset{\frown}{DG}$, $m\angle AEF$, $m\angle DBG$, $m\angle DCA$, and $m\angle CDF$

14 In the accompanying diagram, $\overline{PD}$ is tangent to circle O at D, $\overline{PAC}$ is a secant, chords $\overline{BD}$ and $\overline{AC}$ intersect at E, chord $\overline{AD}$ is drawn, $m\overset{\frown}{BC} = m\overset{\frown}{CA}$, $m\overset{\frown}{BC}$ is twice $m\overset{\frown}{AB}$, and $m\angle DAC = 48$.

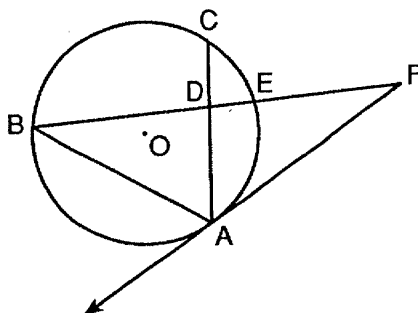Find $m\overset{\frown}{AB}$, $m\overset{\frown}{AD}$, $m\angle CPD$, $m\angle CED$ and $m\angle ADP$.

15 In the accompanying diagram, $\overline{AB} \parallel \overline{CD}$, $\overline{AD}$ and $\overline{DC}$ are tangent to circle O, $m\overarc{AB} = 100$, and $m\overarc{AC} = m\overarc{CB}$.

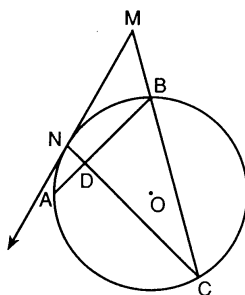Find $m\overarc{AC}$, $m\angle B$, $m\angle D$ and $m\angle BCD$
Is $ABCD$ a parallelogram? [Explain your answer.]

16 In circle O, $\overrightarrow{FA}$ is a tangent, $\overline{FEDB}$ is a secant, $\overline{ADC}$ and $\overline{AB}$ are chords, $m\overarc{CE} = 40$, $m\overarc{AB} = 130$, and $m\angle CAB = 60$.

Find: $m\overarc{BC}$, $m\angle EBA$, $m\angle ADE$, $m\angle F$, $m\angle FAC$

17 In the accompanying diagram of circle O, the ratio $m\overarc{BC} : m\overarc{CA} : m\overarc{AN} : m\overarc{NB}$ is $5:4:1:2$.
Chord $\overline{CB}$ is extended to external point M, chords $\overline{AB}$ and $\overline{CN}$ intersect at D, and tangent $\overrightarrow{MN}$ is drawn.

Find: $m\overarc{BC}$, $m\angle ABC$, $m\angle NMC$, $m\angle NDA$, $m\angle MND$
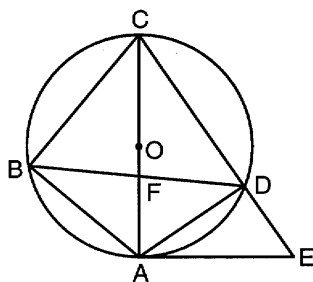
18 In the accompanying diagram, chords $\overline{RT}$ and $\overline{US}$ intersect at Q, secants $\overline{PUT}$ and $\overline{PRS}$ are drawn, $m\overparen{RS} = 120$, $m\overparen{UT} = 80$, $m\angle TRS = 50$, and $\overline{VW}$ is tangent to the circle at T.
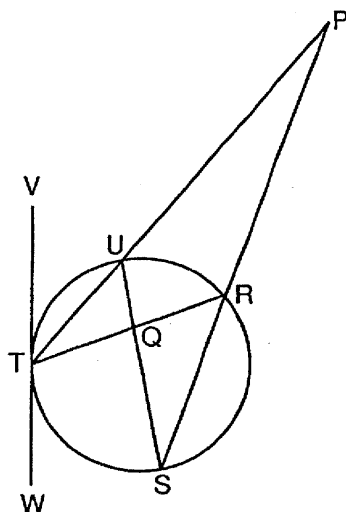
Find $m\overparen{UR}$, $m\angle SUT$, $m\angle P$, $m\angle RQS$ and $m\angle PTV$.
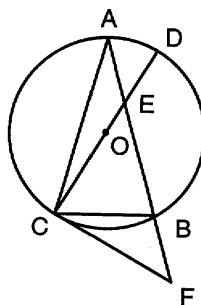

19 In the accompanying diagram of circle O, diameter $\overline{CA}$ intersects chord $\overline{BD}$ at F; $\overline{AE}$ is a tangent; $\overline{EDC}$ is a secant, $\overline{CB}$, $\overline{BA}$, and $\overline{AD}$ are chords; $m\overparen{BC} = 100$; and $m\overparen{AD} = 70$.

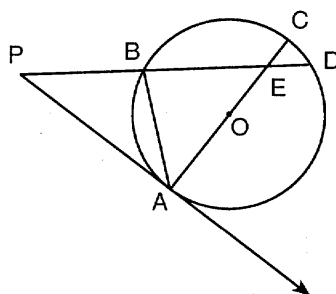Find: $m\overparen{AB}$, $m\angle AEC$, $m\angle BCA$, $m\angle DFA$, $m\angle DAE$.

Name: _____

20 In the accompanying diagram of circle O with inscribed isosceles triangle ABC, $\overline{AB} \cong \overline{AC}$, $m\overset{\frown}{CB} = 60$, $\overline{FC}$ is a tangent, and secant $\overline{FBA}$ intersects diameter $\overline{CD}$ at E.

Find: $m\angle ADC$, $m\overset{\frown}{AD}$, $m\angle DEB$, $m\angle AFC$, $m\angle BCF$
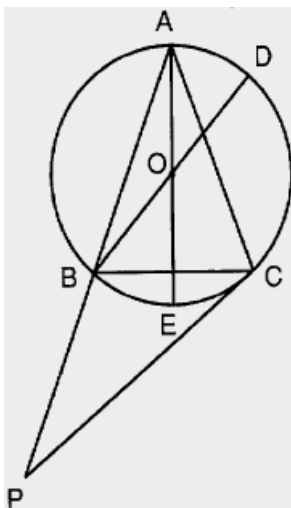
21 In the accompanying diagram, $\overrightarrow{PA}$ is tangent to circle O at point A, secant $\overline{PBD}$ intersects diameter $\overline{AC}$ at point E, chord $\overline{AB}$ is drawn, $m\angle P = 40$, and $m\overset{\frown}{CD}:m\overset{\frown}{DA} = 1:8$.

Find: $m\overset{\frown}{DA}$, $m\overset{\frown}{AB}$, $m\angle BEA$, $m\angle BAC$, $m\angle PBA$.
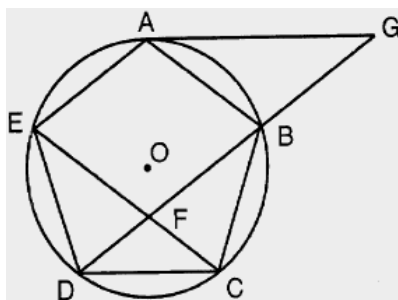
Name: _____

22 In the accompanying diagram, isosceles triangle *ABC* is inscribed in circle *O*, and vertex
    angle *BAC* measures 40º. Tangent $\overline{PC}$, secant $\overline{PBA}$, and diameters $\overline{BD}$ and $\overline{AE}$ are drawn.

Find: $\text{m}\overset{\frown}{BC}$, $\text{m}\angle ABD$, $\text{m}\angle DOE$, $\text{m}\angle P$, $\text{m}\angle ACP$.
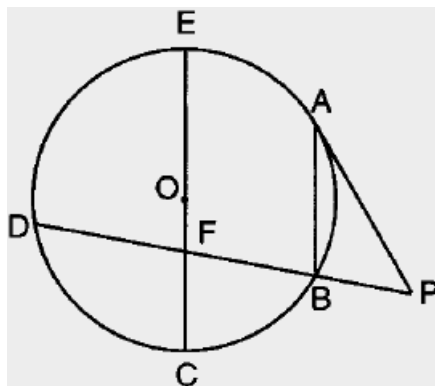
23 In the accompanying diagram, regular pentagon *ABCDE* is inscribed in circle *O*, chords
    $\overline{EC}$ and $\overline{DB}$ intersect at *F*, chord $\overline{DB}$ is extended to *G*, and tangent $\overline{GA}$ is drawn.

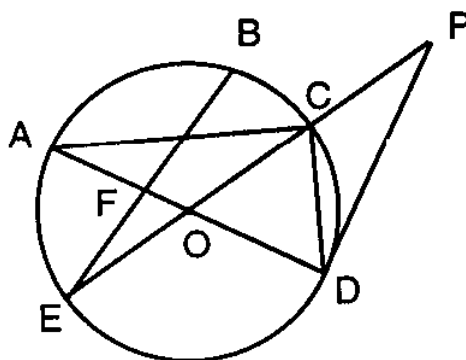Find: $\text{m}\angle BDE$, $\text{m}\angle BFC$, $\text{m}\angle AGD$

24 In the accompanying diagram of circle O, chord $\overline{AB}$ is parallel to diameter $\overline{EC}$, secant $\overline{PBD}$ intersects $\overline{EC}$ at F, tangent $\overline{PA}$ is drawn, $m\overset{\frown}{AB} = m\overset{\frown}{BC}$, and $m\overset{\frown}{CD} = 80$.

Find: $m\overset{\frown}{AE}$, $m\angle ABD$, $m\angle DFC$, $m\angle P$, $m\angle PAB$.

25 In the accompanying diagram of circle O, diameter $\overline{EOC}$ is extended through C to point P; diameter $\overline{AFOD}$, tangent $\overline{PD}$, and chords $\overline{AC}$, $\overline{CD}$, $\overline{BFE}$ are drawn; $m\angle COD = 60$; and $m\angle AFB = 100$.

Find: $m\overset{\frown}{DE}$; $m\angle P$; $m\angle ACE$, $m\overset{\frown}{AB}$, $m\angle ACD$.
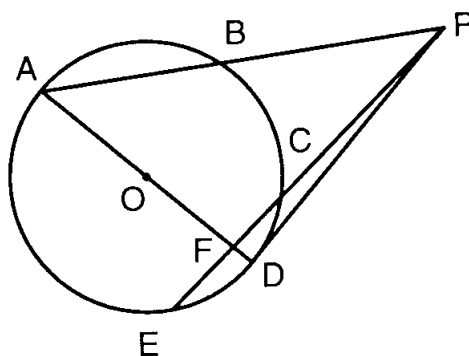
Name: _____

26 In the accompanying diagram of circle O, $m\widehat{AB}:m\widehat{BC} = 1:2$; diameter $\overline{CA}$ and chord $\overline{AE}$ are drawn; chord $\overline{EC}$ is parallel to chord $\overline{AB}$; chord $\overline{BC}$ is extended through C to D; and tangent $\overline{DE}$ is drawn.
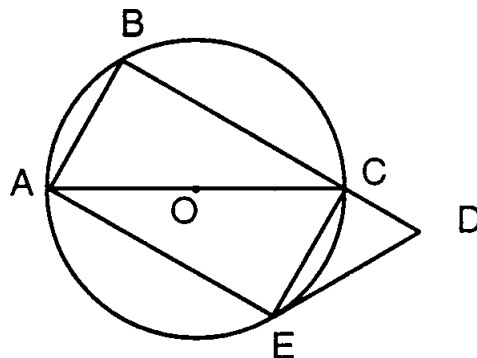
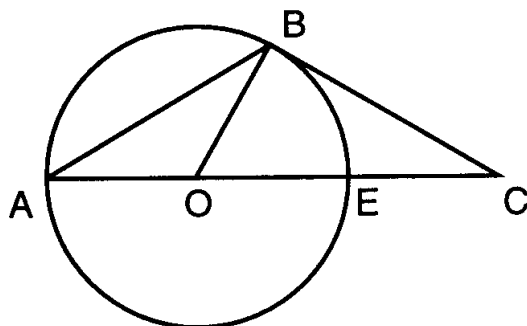Find: $m\widehat{BC}$, $m\widehat{CE}$, $m\angle AEC$, $m\angle CED$, $m\angle BDE$.

27 In the accompanying diagram of circle O, $m\widehat{AC} = 140$, $m\widehat{AE} = 130$, $m\widehat{AB}:m\widehat{BC} = 6:4$, $\overline{PD}$ is a tangent, secant $\overline{PCE}$ intersects diameter $\overline{AD}$ at F, and secant $\overline{PBA}$ is drawn.

Find $m\widehat{ED}$, $m\widehat{AB}$, $m\angle BAD$, $m\angle APE$, $m\angle EFD$
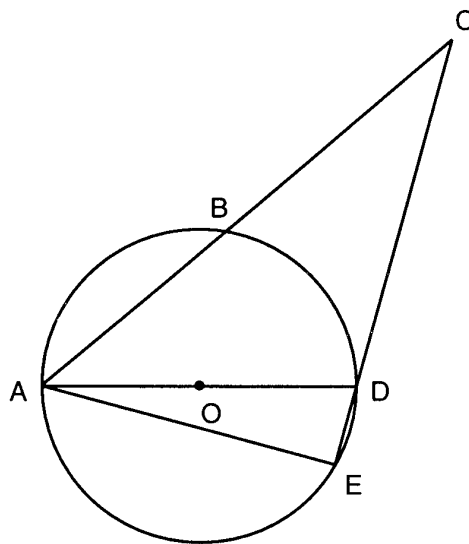
Name: _____

28 In the accompanying diagram of circle *O*, diameter $\overline{AE}$ is extended through *E* to *C*; tangent
$\overline{CB}$, chord $\overline{AB}$, and radius $\overline{OB}$ are drawn; and $m\overparen{AB}:m\overparen{BE} = 2:1$.

a Find: $m\overparen{AB}$, m∠*BAC*, m∠*C*, m∠*ABC*.
b Is ▲*OBC* acute, right, obtuse or equiangular? Explain your answer.
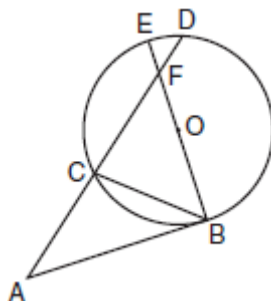

29 In the accompanying diagram of circle *O*, diameter $\overline{AD}$, chord $\overline{AE}$, and secants $\overline{CBA}$ and
$\overline{CDE}$ are drawn; m∠*BAD* = 40; and $m\overparen{AE} = 5(m\overparen{ED})$.

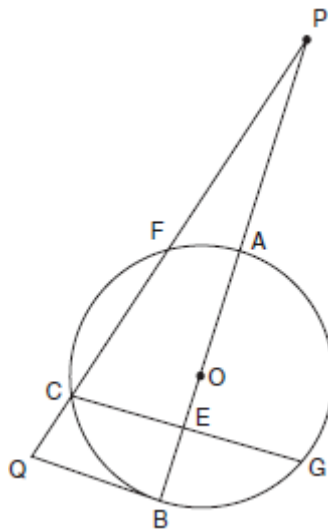Find: $m\overparen{BD}$, $m\overparen{AE}$, m∠*ACE*, m∠*AED*, m∠*ADC*.

Name: _____

30 In the accompanying diagram of circle O, tangent $\overline{AB}$ and chord $\overline{BC}$ are drawn, secant $\overline{ACD}$ intersects diameter $\overline{EB}$ at F, $m\overarc{BD} = 160$, and $m\overarc{BC} = 80$.

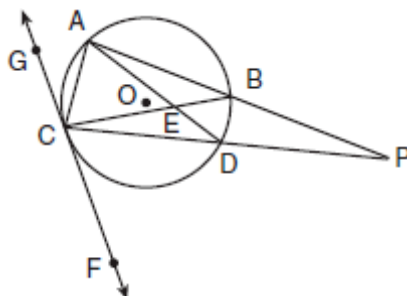Find: $m\angle A$, $m\angle ABE$, $m\angle ABC$, $m\angle EFC$, $m\angle ACB$

31 In the accompanying diagram of circle O, secant $\overline{PFCQ}$, secant $\overline{PAOEB}$, tangent $\overline{QB}$, and chord $\overline{CEG}$ are drawn; $m\overarc{BC}:m\overarc{CF}:m\overarc{FA} = 7:8:3$; and $m\angle AEG = 95$.

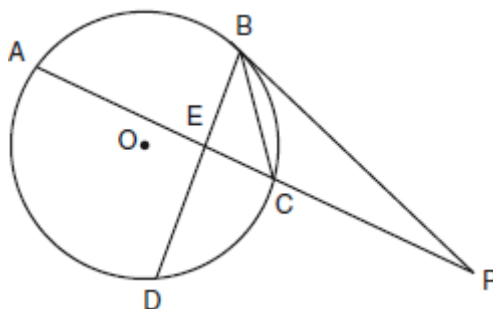Find: $m\overarc{CF}$, $m\overarc{AG}$, $m\angle P$, $m\angle FCG$, $m\angle FQB$

Name: _____

32 In the accompanying diagram of circle O, secant $\overline{ABP}$, secant $\overline{CDP}$, and chord $\overline{AC}$ are drawn; chords $\overline{AD}$ and $\overline{BD}$ intersect at E, tangent $\overleftrightarrow{GCF}$ intersects circle O at C, and $m\overparen{AB}:m\overparen{BD}:m\overparen{DC}:m\overparen{CA} = 8:2:5:3$.
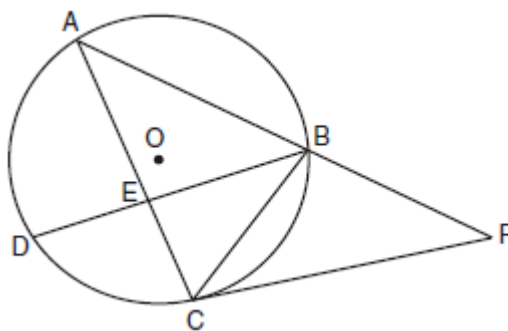

Find: $m\overparen{CA}$, $m\angle ACB$, $m\angle P$, $m\angle AEB$, $m\angle DCF$

33 In the accompanying diagram of circle O, tangent $\overline{PB}$, secant $\overline{AECP}$, chord $\overline{DEB}$, and chord $\overline{CB}$ are drawn; $m\overparen{DC} = 90$; $m\angle DEC = 85$; $BP = 15$; and $CB = 8$.

Find: $m\overparen{AB}$; $m\angle ACB$; $m\angle P$ to the *nearest degree*.
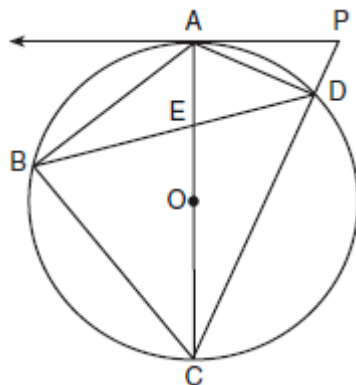
34 In the accompanying diagram of circle O, chords $\overline{BD}$, $\overline{BC}$, and $\overline{AC}$, tangent $\overline{PC}$, and secant $\overline{ABP}$ are drawn; $m\angle DBC = 40$, $m\angle AEB = 110$; and $m\overparen{AD}:m\overparen{CB} = 9:5$.

Find: $m\overparen{AB}$, $m\overparen{AD}$, $m\angle P$, $m\angle BCP$, $m\angle ACP$

Name: _____

35 In the accompanying diagram of circle O, $\overrightarrow{PA}$ is tangent to the circle at A; $\overline{PDC}$ is a secant;
diameter $\overline{AEOC}$ intersects chord $\overline{BD}$ at E; chords $\overline{AB}$, $\overline{BC}$, and $\overline{DA}$ are drawn; $m\overset{\frown}{DA} = 46$;
and $m\overset{\frown}{BC}$ is 32 more than $m\overset{\frown}{AB}$.

Find: $m\overset{\frown}{AB}$; $m\angle BAC$; $m\angle P$; $m\angle DEC$; $m\angle PDA$